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A COMPUTATIONAL IMPLEMENTATION FOR TRANSIENT TEMPERATURE FIELD ANALYSIS AROUND A CASED AND CEMENTED WELLBORE BASED ON ANALYTICAL SOLUTION

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Resumo –.A estimativa do campo de temperatura em torno do poço é um importante problema na engenharia de petróleo. O calor é transferido entre o poço e a formação durante os processos de recuperação térmica, induzindo tensões que podem afetar a estabilidade estrutural do poço e da formação. O campo de temperatura em torno do poço tem sido avaliado por técnicas numéricas como o método das diferenças finitas ou inversão numérica da transformada de Laplace, supondo uma temperatura constante na parede do poço. O método aqui proposto considera a condução radial do calor através da parede do poço, sendo este composto por múltiplas camadas cilíndricas de materiais distintos com propriedades físicas constantes e em perfeito contato térmico. A solução para a equação de condução utiliza o método da separação de variáveis, através das funções de Bessel. O domínio do problema é definido num intervalo finito limitado pelo raio de influência, avaliado conforme o avanço da frente de calor. A condição de contorno na parede do poço é dada pela taxa transferência de calor. A implementação computacional desta solução apresentou um bom desempenho em termos de tempo de processamento e simplicidade do modelo.

Palavras-Chave: Injeção de vapor, Condução de calor, Reservatórios de óleo pesado

Abstract – Estimation of the temperature field around a wellbore is an important problem in petroleum engineering. Heat is transferred between the borehole and the formation during thermal recovery techniques, inducing stresses that may affect the structural wellbore and formation stability. Temperature field around the well is usually evaluated by numerical techniques such as the finite difference method or numerical inversion of Laplace transform, assuming a constant temperature at the borehole wall. The method proposed herein considers the radial heat conduction through the bore-face. Cylindrical multilayer of heterogeneous materials in perfect thermal contact and with constant physical properties composes the geometry of the problem. The heat conduction equation is evaluated through the separation-of-variables method, using Bessel functions. The problem domain is defined at the finite interval bounded by the radius of thermal influence, evaluated as the heat front advances. At the borehole wall, the boundary condition is given by a constant heat transfer rate. The solution implemented in a computer program presented a good computational performance in terms of time and model simplicity.

Keywords: Steam injection, Heat conduction, Heavy oil reservoirs

1. Introduction

Estimation of the temperature field around a wellbore is an important problem in petroleum engineering. Heat is transferred between the borehole and the formation during thermal recovery techniques, inducing stresses that may affect the structural wellbore and formation stability. The thermal stresses generated by high temperature changes may cause material damage, such as collapse, buckling or shear failure of the casing and hydraulic sealing loss of the cement sheath due to its cracking. High temperatures in open-hole completions may lead to grain shearing or cracking, increasing effective borehole radius. The evaluation of the stress state induced by temperature changes is of major interest in oil wellbore drilling and exploitation.

Numerical methods for the thermo-elastic stresses around a multi-layered cylinder were proposed by Kandil et al. (1995), Jane and Lee (1999), Hung et al. (2001). Many numerical simulators are available to define the temperature field around a wellbore. They can deal with different operational conditions of the well and include the heat transfer through any kind of geometry. However, numerical simulations are often complicated for field applications because usually the required material properties are not well known.

The method proposed herein is based on the work of de Monte (2002), that presented an analytical solution to the unsteady heat conduction problem for multi-layered solids with any geometry. It is assumed that heat is conducted from the bore-face to the formation through multilayer cylindrical of heterogeneous materials in perfect thermal contact and with constant physical properties composes the geometry of the problem. The unsteady heat conduction equation is evaluated through the separation-of-variables method, using Bessel functions. The problem domain is defined at the finite interval bounded by the radius of thermal influence, evaluated as the heat front advances. At the borehole wall, the boundary condition is given by a constant heat transfer rate. Given the temperature field, the stress state surrounding the borehole is evaluated through the thermo-elasticity theory.

2. Mathematical Model

The mathematical model for heat conduction in a wellbore subject to thermal recovery, showed in Figure 1, has the following assumptions:

i. Heat flux is radial through the various layers that compose the wellbore completion until a radius of thermal influence inside the rock.
ii. The radius of thermal influence is the distance from the borehole axis at which the heat flux may be considered as null.
iii. The radius of thermal influence varies with time according to the system heating, that is, $r_6(t)$.
iv. Each cylindrical layer has homogeneous and time independent thermal properties. They represent the different materials that compose the system (production casing, production casing cement sheath, external casing, external casing cement sheath and rock).
v. There is no heat generation inside the system.
vi. The temperature T_∞ of the fluid inside the production casing is spatially uniform and is constant along the time.
vii. The convective heat transfer coefficient h from the fluid to the production casing is uniform and constant.
viii. The cylindrical layers are in perfect thermal contact, what means that heat flux and temperatures are continuous at the surface contacts.

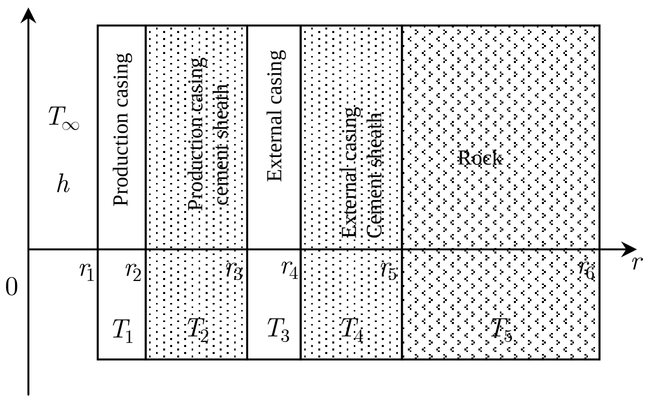


Figure 1. Scheme of a wellbore with double casing

According to the assumptions, the transient unidimensional heat flow is given by the following partial differential equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_i r \frac{\partial T_i}{\partial r}\right) = \rho_i c_{p_i} \frac{\partial T_i}{\partial t},\ r \in [r_1, r_6]\ (i = 1,2,...,5). \tag{1}$$

Based on de Monte (2002) work, who developed a mathematical model for transient unidimensional heat flow for systems of composite media, and defining $\theta_i(r,t) = T_\infty - T_i(r,t)\ \ (i = 1,2,...,5)$, a final mathematical formulation in cylindrical coordinates is obtained:

Partial differential equation for heat conduction:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \theta_i}{\partial r}\right) = \frac{1}{\alpha_i}\frac{\partial \theta_i}{\partial t},\ r \in [r_1, r_6]\ (i = 1,2,...,5),$$
$$\alpha_i = \frac{k_i}{\rho_i c_{p_i}},\ (t \geq 0). \tag{2}$$

Boundary condition at bore-face $(r = r_1)$:

$$-k_1\left(\frac{\partial \theta_1}{\partial r}\right)_{r_1} + h\theta_1(r_1, t) = 0. \tag{3}$$

Boundary condition at intermediary faces $(r = r_i)$:

$$\theta_{i-1}(r_i, t) = \theta_i(r_i, t),\ (i = 2,3,4,5), \tag{4}$$

$$k_{i-1}\left(\frac{\partial \theta_{i-1}}{\partial r}\right)_{r_i} = k_i\left(\frac{\partial \theta_i}{\partial r}\right)_{r_i}\ \ (i = 2,3,4,5). \tag{5}$$

Boundary condition at the radius of thermal influence: $(r = r_6)$:

$$-k_5\left(\frac{\partial \theta_5}{\partial r}\right)_{r_6} = 0. \tag{6}$$

Initial conditions:

$$\theta_i(r, t = 0) = \theta_0,\ r \in [r_1, r_6]\ (i = 1,2,...,5). \tag{7}$$

The analytical technique developed by de Monte (2002) is applied for the solution of the set of equations (2-7). In this case, the external boundary condition, given by equation (6), substitutes the external boundary condition presented by the mathematical model from de Monte.

3. Analytical Solution for heat conduction at the wellbore

This section presents the analytic technique for the solution of transient heat conduction in the wellbore. In this technique, a set of normalized variable substitutes the physical parameters of the problem. The dimensionless temperature field in each layer of the system is described by:

$$\Theta_i(\xi, \tau) = \sum_{m=1}^{\infty} c_m \Phi_{i,m} \mathrm{X}_{i,m}(\xi) e^{-\beta_m^2 \tau},$$
$$\tau \geq 0,\ \xi \in [\gamma_i, \gamma_{i+1}]\ (i = 1,2,...,5). \tag{8}$$

where Fourier number $\tau = \alpha_1 t / r_1^2$ is the dimensionless time. The coefficients c_m are defined by the following equation:

$$c_m = \frac{1}{N_m \beta_m} \sum_{i=1}^{5} \Phi_{i,m}\left(\frac{k_i}{\sqrt{\delta_i}}\right)\left[\xi \Lambda_{i,m}(\xi)\right]_{\gamma_i}^{\gamma_{i+1}}, \tag{9}$$

where the normalization constant N_m associated to each eigenvalue-valor β_m is given by:

$$N_m = \frac{1}{2}\sum_{i=1}^{5}\left(\Phi_{i,m}\right)^2\left(\frac{k_i}{\sqrt{\delta_i}}\right)\left\{\left[\xi \mathrm{X}_{i,m}(\xi)\right]^2 + \left[\xi\Lambda_{i,m}(\xi)\right]^2\right\}_{\gamma_i}^{\gamma_{i+1}} \tag{10}$$

The constant $\Phi_{i,m}$ associated to the eigenvalue β_m in each cylindrical layer is evaluated according to the following expressions:

$$\begin{aligned}
\Phi_{1,m} &= 1\\
\Phi_{2,m} &= \frac{\mathrm{X}_{1,m}(\gamma_2)}{\mathrm{X}_{2,m}(\gamma_2)}\\
\Phi_{3,m} &= \frac{\mathrm{X}_{1,m}(\gamma_2)\mathrm{X}_{2,m}(\gamma_3)}{\mathrm{X}_{2,m}(\gamma_2)\mathrm{X}_{3,m}(\gamma_3)}\\
\Phi_{4,m} &= \frac{\mathrm{X}_{1,m}(\gamma_2)\mathrm{X}_{2,m}(\gamma_3)\mathrm{X}_{3,m}(\gamma_4)}{\mathrm{X}_{2,m}(\gamma_2)\mathrm{X}_{3,m}(\gamma_3)\mathrm{X}_{4,m}(\gamma_4)}\\
\Phi_{5,m} &= \frac{\mathrm{X}_{1,m}(\gamma_2)\mathrm{X}_{2,m}(\gamma_3)\mathrm{X}_{3,m}(\gamma_4)\mathrm{X}_{4,m}(\gamma_5)}{\mathrm{X}_{2,m}(\gamma_2)\mathrm{X}_{3,m}(\gamma_3)\mathrm{X}_{4,m}(\gamma_4)\mathrm{X}_{5,m}(\gamma_5)}
\end{aligned} \tag{11}$$

The equations that describe the eigenfunctions $\mathrm{X}_{i,m}(\xi)$ and the normalization functions $\Lambda_{i,m}(\xi)$ $(i = 1,2,...,5)$ in equations (8), (9) and (10) correspond to each eigenvalue β_m and are described by equation (12) and (13), respectively:

$$\mathrm{X}_{i,m}(\xi) = J_0\left(\frac{\beta_m\xi}{\sqrt{\delta_i}}\right) + \Pi_{i,m}Y_0\left(\frac{\beta_m\xi}{\sqrt{\delta_i}}\right), \tag{12}$$

$$\begin{aligned}
&\Lambda_{i,m}(\xi) = J_1\left(\frac{\beta_m\xi}{\sqrt{\delta_i}}\right) + \Pi_{i,m}Y_1\left(\frac{\beta_m\xi}{\sqrt{\delta_i}}\right),\\
&\xi \in [\gamma_i, \gamma_{i+1}]\ (i = 1,2,...,5),
\end{aligned} \tag{13}$$

where J_0, J_1, Y_0, Y_1 are zero-order Bessel functions of the first kind, first-order Bessel function of the second kind, zero-order Bessel function of the second kind, and first-order Bessel functions of second kind, respectively. The ariable $\xi = r / r_1$ is the dimensionless space coordinate and $\delta_i = \alpha_i / \alpha_1\ (i = 1,2,...,5)$ is the thermal diffusivity ratio for each cylindrical layer relative to the production casing. The constants $\Pi_{i,m}\ (i = 1,2,...,5)$ that correspond to each eigenvalue β_m are evaluated by:

$$\Pi_{1,m} = -\frac{BiJ_0(\beta_m) + \beta_m J_1(\beta_m)}{BiY_0(\beta_m) + \beta_m Y_1(\beta_m)}, \tag{14}$$

$$\begin{aligned}
\Pi_{i,m} = &-\Bigg[\left(\frac{\kappa_i}{\sqrt{\delta_i}}\right)J_1\left(\frac{\beta_m\gamma_i}{\sqrt{\delta_i}}\right)\mathrm{X}_{i-1,m}(\gamma_i)\\
&-\left(\frac{\kappa_{i-1}}{\sqrt{\delta_{i-1}}}\right)J_0\left(\frac{\beta_m\gamma_i}{\sqrt{\delta_i}}\right)\Lambda_{i-1,m}(\gamma_i)\Bigg]\\
&\Bigg/\Bigg[\left(\frac{\kappa_i}{\sqrt{\delta_i}}\right)Y_1\left(\frac{\beta_m\gamma_i}{\sqrt{\delta_i}}\right)\mathrm{X}_{i-1,m}(\gamma_i)\\
&-\left(\frac{\kappa_{i-1}}{\sqrt{\delta_{i-1}}}\right)Y_0\left(\frac{\beta_m\gamma_i}{\sqrt{\delta_i}}\right)\Lambda_{i-1,m}(\gamma_i)\Bigg]\ (i = 2,3,4),
\end{aligned} \tag{15}$$

$$\Pi_{5,m} = -\frac{\left(\frac{\beta_m\kappa_5}{\sqrt{\delta_5}}\right)J_1\left(\frac{\beta_m\gamma_6}{\sqrt{\delta_5}}\right)}{\left(\frac{\beta_m\kappa_5}{\sqrt{\delta_5}}\right)Y_1\left(\frac{\beta_m\gamma_6}{\sqrt{\delta_5}}\right)} \tag{16}$$

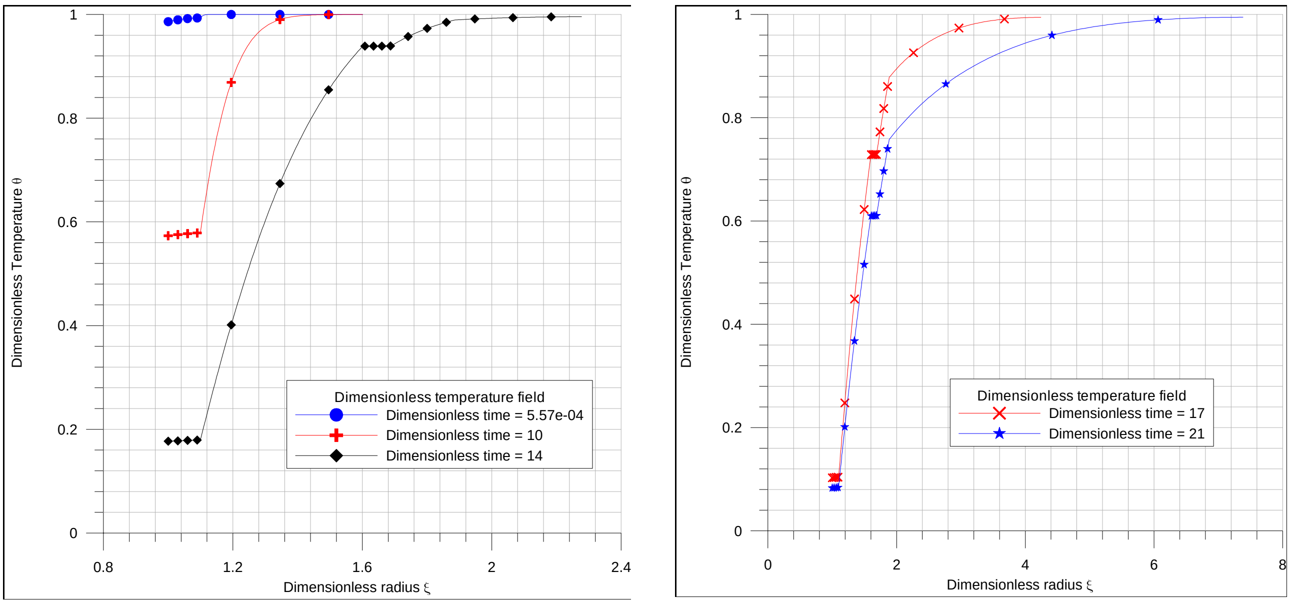
The dimensionless eigenvalue β_m is the m-th root of the following transcendental equation:

$$\frac{\left[\left(\frac{\kappa_5}{\sqrt{\delta_5}}\right)J_1\left(\frac{\beta\gamma_5}{\sqrt{\delta_5}}\right)\mathrm{X}_4(\gamma_5)-\left(\frac{\kappa_4}{\sqrt{\delta_4}}\right)J_0\left(\frac{\beta\gamma_5}{\sqrt{\delta_5}}\right)\Lambda_4(\gamma_5)\right]}{\left[\left(\frac{\kappa_5}{\sqrt{\delta_5}}\right)Y_1\left(\frac{\beta\gamma_5}{\sqrt{\delta_5}}\right)\mathrm{X}_4(\gamma_5)-\left(\frac{\kappa_4}{\sqrt{\delta_4}}\right)Y_0\left(\frac{\beta\gamma_5}{\sqrt{\delta_5}}\right)\Lambda_4(\gamma_5)\right]}
-\frac{\left[\left(\frac{\beta\kappa_5}{\sqrt{\delta_5}}\right)J_1\left(\frac{\beta\gamma_6}{\sqrt{\delta_5}}\right)\mathrm{X}_4(\gamma_5)\right]}{\left[\left(\frac{\beta\kappa_5}{\sqrt{\delta_5}}\right)Y_1\left(\frac{\beta\gamma_6}{\sqrt{\delta_5}}\right)\mathrm{X}_4(\gamma_5)\right]}=0. \tag{17}$$

Since the solution of the temperature field given by equation (8) is an infinite sum, ths serie is truncated when the solution achieves a convenient precision. According to de Monte, using 30 terms in the sum of equation (8) the approximate solution presents an error less than 2.8 %. In this work, the implemented algorithm do not fix the number of terms of the series, which may be changed according to the application. The equation **(17)** is solved numerically by the bisection method described by Heath (1997). The search for the roots follows a criteria of finding the smallest value to higher vallues through successive intervals where there are signal changes in the value of function **(17)**.

4. Case Study

The example preseneted herein consists of a wellbore with double casing in a limestone formation. The assumed wellbore diameter is 12”, the production casing has 7” and 23 nominal weight, the external casing has 10.75” with 32.75 nominal weight. The thermal parameters for thermal conductivity are 45.10 W/m-ºC (steel), 0.38 W/m-ºC (cement sheath) and 1.67 W/m-ºC for rock. The specific heat of the steel is 461.00 J/kg-ºC, for cement is 1100 J/kg-ºC and 900 J/kg-ºC for rock. The specific weight for the three materials are 7850, 1993 and 2560 kg/m^3 for steel, cement and rock, respectively. The figures 2 (a)-(b) present the temperature field obtained for various time steps: This temperature fields were validated using ANSYS finite element code, with good agreement.
The figure 2-(a) show some of the initial time steps and figure 2-(b) show two of the last ones. Comparing both figures, it may be seen that along the time, the thermal radius of influence increases as the heat front advances. The contranst between the thermal properties of the materials that compose the system can be seen by the different gradient of the curves presented.



(a) (b)

Figure 2. Temperature field (a) for the first time steps and (b) for last time steps

5. Conclusions

The method presented herein is a simple method to solve the transient heat conduction equation in cylinders composed of various materials, with application to the evaluation of temperature field around cased and cemented wellbores subjected to thermal recovery methods. This formulation implemented in a computer colde is rather faster and simpler than the numerical methods usually applied to this problem.

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7. Nomenclature

Bi=*Biot numbers at the outer boundary surfacs:* hx_1/k_1	β_m=*mth dimensionless eigenvalue*
c=*integration coefficient*	γ_i=*geometric ratio:* $r_i = r_1$ *(i= 1; 2;…;6)*
h=*convective heat transfer coefficient*	δ_i=*thermal diffusivity ratio:* $\alpha_i = \alpha_1$ *(i= 1; 2;…;5)*
J_0=*zero-order Bessel function of the first kind*	θ_i=*temperature difference for the ith laye:* $T_\infty - T_i$
J_1=*first-order Bessel function of the first kind*	θ_0=*uniform initial temperature difference:* $T_\infty - T_0$
K_i=*thermal conductivity of the ith cylindrical layer*	Θ_i=*dimensionless temperature for the ith layer:* θ_i / θ_0
t=*time*	k_i=*thermal conductivity ratio:* K_i/K_1 *(i= 1; 2;…;5)*
T_i=*temperature for the ith layer of the sistem*	ξ=*dimensionless space coordinate:* r/r_1
T_∞=*fluid temperature*	Π_i=*functions defined by Eqs.(14)-(16)*
r_i=*values of the space coordinate at the boundary*	τ=*dimensionless time:* $\tau = \alpha_1 t / r_1^2$
X_i=*mth eigenfunction corresponding to* β_m	$\Phi_{i,m}$=*functions defined by Eqs. (11)*
Y_0=*zero-order Bessel function of the second kind*	
Y_1=*first-order Bessel function of the second kind*	
α_I=*thermal diffusivity of the ith layer*	

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